A Generalized Earley Parser for Human Activity Parsing and Prediction

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Abstract—Detection, parsing, and future predictions on sequence data (e.g., videos) require the algorithms to capture non-Markovian and compositional properties of high-level semantics. Context-free grammars are natural choices to capture such properties, but traditional grammar parsers (e.g., Earley parser) only take symbolic sentences as inputs. In this paper, we generalize the Earley parser to parse sequence data which is neither segmented nor labeled. Given the output of an arbitrary probabilistic classifier, this generalized Earley parser finds the optimal segmentation and labels in the language defined by the input grammar. Based on the parsing results, it makes top-down future predictions. The proposed method is generic, principled, and wide applicable. Experiment results clearly show the benefit of our method for both human activity parsing and prediction on three video datasets.

Index Terms—video understanding, high-level vision, activity recognition, activity prediction, grammar models, grammar parser

1 INTRODUCTION

We propose an algorithm to tackle the task of understanding complex human activities from (partially-observed) videos from two important aspects: activity recognition and prediction. This is a ubiquitous problem driven by a wide range of applications in many perceptual tasks. Some scenarios further require the algorithm to have both recognition and prediction capabilities, e.g., assistive robots would need to recognize the current human activity and provide future-aware assistance.

To find a joint solution of activity recognition and prediction, we need to consider two questions: 1) what is a good representation for the structure of human activities/tasks, and 2) what is a good inference algorithm to cope with such a representation. A popular family of representations for events is the Markov models (e.g., hidden Markov Model). However, Markov models are not expressive enough since human tasks often exhibit non-Markovian and compositional properties. Hence we argue that 1) a representation should reflect the hierarchical/compositional task structure of long-term human activities, and 2) an inference algorithm should recover the hierarchical structure given the past observations, and be able to predict the future.

We refer to the Chomsky hierarchy to choose a model to capture the hierarchical structure of the entire history. The Chomsky hierarchy is a containment hierarchy of classes of formal grammars in the formal languages of computer science and linguistics. The reason is that activities are analogous to languages: actions are like words and activities are like languages. The Chomsky hierarchy categorizes language models into four levels: 1) Turing machines, 2) context-sensitive grammars, 3) context-free grammars, and 4) regular grammars. Higher-level models contain lower-level models, and Markov models belongs to the lowest level (regular grammars). In this paper, we propose to use context-free grammars to parse and predict human activities. In the definition of formal language theory, a grammar is a set of production rules for sentences in a formal language. In our case, the rules describe how to form sentences (activities) from the language’s alphabet (actions) that are valid.

However, it has not been possible to directly use symbolic grammars to parse and label sequence data (e.g., videos). Traditional grammar parsers take symbolic sentences as inputs instead of noisy sequence data. The data has to be i) segmented and ii) labeled to be parsed by existing...
grammar parsers. One naive solution is to first segment and label the data using a detector and thus generating a label sentence. Then grammar parsers can be applied on top of it for parsing prediction. But this is apparently non-optimal, since the grammar rules are not considered in the detection/classification process. It may not even be possible to parse this label sentence, because the output from detectors are very often grammatically incorrect.

In this paper, we design a grammar-based parsing algorithm that directly operates on input sequence data, which goes beyond the scope of symbolic string inputs for classic parsing algorithms. Specifically, we propose a generalized Earley parser to take \textit{probabilistic sequence inputs} instead of deterministic symbolic inputs, based on the classic Earley parser [1]. The algorithm finds the optimal segmentation and label sentence according to both a symbolic grammar and a classifier output of probabilities of labels for each frame as shown in Figure 1. Optimality here means maximizing the joint probability of the label sentence according to the grammar prior and classifier output while being grammatically correct.

The difficulty of achieving this optimality lies in the joint optimization of both the grammatical structure and the parsing likelihood of the output label sentence. For example, an expectation-maximization-type of algorithm will not work well since i) there is no guarantee for optimality, and ii) any grammatically incorrect sentence has a grammar prior of probability 0. The algorithm can easily get stuck in local minimums and fail to find the optimal solution that is grammatically correct.

The core idea of our algorithm is to directly and efficiently search for the optimal label sentence in the language defined by the grammar. The constraint of the search space ensures that the sentence is grammatically correct. Specifically, a heuristic search is performed on the prefix tree expanded according to the grammar, where the path from the root to a node represents a partial sentence (prefix). We search through the prefixes to find the best sentence according to a heuristic. By carefully defining the heuristic as a prefix probability computed based on the grammar prior and classifier output, we can efficiently search through the tree to find the optimal label sentence.

The generalized Earley parser has four major advantages. i) The inference process highly integrates a high-level grammar with an underlying classifier; the grammar gives guidance for segmenting and labeling the sequence data and future predictions. ii) The only requirement for the underlying classifier is that the classifier should give probabilistic outputs. This makes the algorithm widely applicable, since almost all statistical learning classifiers are probabilistic. iii) It generates semantically meaningful results (a grammar parse tree) for data sequence, and the process is highly explainable. iv) It is principled and generic, as it applies to most sequence data parsing and prediction problems (the data does not have to be videos).

We evaluate the proposed approach on three datasets of human activities in the computer vision domain. The first dataset CAD-120 [2] consists of daily activities and most activity prediction methods are based on this dataset. Comparisons show that our method significantly outperforms state-of-the-art methods on future activity prediction. The second dataset Watch-n-Patch [3] is designed for “action patching”, which includes daily activities that have action forgotten by people. Experiments on the second dataset show the robustness of our method on noisy data. The third dataset Breakfast [4] consists long videos of daily cooking activities. Results on this dataset show comparisons between our method and other structured modeling and language-inspired modeling methods. All experiments show that the generalized Earley parser performs well on both activity parsing and prediction tasks.

This paper makes three major contributions.

- It designs a parsing algorithm for symbolic context-free grammars that directly operates on sequence data. It can obtain the optimal segmentation and labels according to the grammar prior and classifier outputs.
- It proposes a prediction algorithm that naturally integrates with this parsing algorithm.
- It formulates an objective for future prediction for both grammar induction and classifier training. The generalized Earley parser serves as a concrete example for combining symbolic reasoning methods and connectionist approaches.

2 Related Work

This paper is an extension of previous ICCV and ICML papers [3, 4]. The extension includes two major aspects: 1) for the method, we have extended the algorithm to incorporate a non-trivial grammar prior into the generalized Earley parser, and 2) in the experiments, we tested the model on more datasets with more comparisons and in-depth analyses.

Activity parsing refers to recognition and segmentation of long-term and complicated activities from videos, whereas action recognition corresponds to short-term actions. They are two extensively-studied topics in computer vision and we refer the readers to a survey [7] for a more comprehensive treatment. The main stream of work on activity recognition is to extend mid-level representations to high-level representations.

These extensions are designed in several different ways to model the complex activity structures. A number of methods have been proposed to model the high-level temporal structure of low-level features extracted from video [8, 9, 10]. Some other approaches represent complex activities as collections of attributes [14, 15, 16, 17]. Another important type of methods builds compositional/hierarchical models on actions [15, 18, 19, 20, 21, 22, 23]. Koppula et al. [2] proposed a model incorporating object affordances that detects and predicts human activities. Wei et al. [24] proposed a 4D human-object interaction model for event recognition. In some recent works, structural models are implicitly learned by neural networks [25, 26, 27, 28, 29].

Grammar models fall into the category of compositional models for temporal structures. Ivanov and Bobick [30] proposed to first generate a discrete symbol stream from continuous low-level detectors, and then applied stochastic context-free parsing to incorporate prior knowledge of the temporal structure. Pei et al. [31] detected atomic actions and used a stochastic context sensitive grammar for video parsing and intent prediction. Similar to the generalized Earley parser, it parses the video in an online fashion and
enables prediction. However, the algorithm uses manually defined thresholds to detect action transitions. Kuehne et al. [4] modeled action units by hidden Markov models (HMMs), and models the higher-level action sequence by context-free grammars. Pirsiavash and Ramanan [32] proposed segmental grammar for video parsing, which extends regular grammars to allow non-terminals to generate a segment of terminals of certain lengths. Vo and Bobick [33] generated a Bayes network, termed Sequential Interval Network (SIN), where the variable nodes correspond to the start and end times of component actions. This network then makes inference about start and end times for detected action primitives. Qi et al. [5] proposed to integrate spatial-temporal attributes to terminal nodes of a context-free grammar. Based on Earley parser, an activity parsing and prediction algorithm is proposed. Overall, grammar-based methods have shown effectiveness on tasks that have compositional structures.

However, the above grammar-based algorithms (except [32]) take symbolic inputs like the traditional language parsers. They require the action primitives/atomic actions to be first detected, then a grammar is used for high-level parsing. This limits the applicability of these algorithms. Additionally, the parser does not provide guidance for either training the classifiers or segmenting the sequences. They also lack a good approach to handle grammatically incorrect label sentences. For example, Qi et al. [5] found in the training corpus the closest sentence to the recognized sentence and applies the language parser afterward. Pirsiavash and Ramanan [32] ensures the results are grammatically correct, but it makes the grammar unnecessarily redundant (each possible segment length will make a new copy for each original grammar rule).

In our case, the proposed parsing algorithm takes sequence data of raw signals and a typical context-free grammar as input. It then generates the label sentence as well as the parse tree. All parsed label sentences are grammatically correct, and a learning objective is formulated for the classifier. Our work also serves as a bridge between connectionist and symbolic approaches, and it does not have any constraint on the low-level classifier.

**Future activity prediction** is a relatively new domain in computer vision. [6, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47] predict human trajectories/actions in various settings including complex indoor/outdoor scenes and crowded spaces. Li and Fu [43] built a probabilistic suffix tree to model the Markov dependencies between action units and thus predict future events using a compositional model. Walker et al. [41] predicted not only the future motions in the scene but also the visual appearances. In some recent work, Koppula and Saxena [48] used an anticipatory temporal conditional random field to model the spatial-temporal relations through object affordances. Jain et al. [49] proposed structural-RNN as a generic method to combine high-level spatial-temporal graphs and recurrent neural networks, which is a typical example that takes advantage of both graphical models and deep learning. Qi et al. [5] proposed a spatial-temporal And-Or graph (ST-AOG) for activity prediction. In this work, we present the generalized Earley parser, in which the recognition and prediction are naturally and tightly integrated.

### 3 Representation: Probabilistic Context-Free Grammars

We model complex activities by grammars, where low-level actions are terminal symbols, i.e., like words in a language. In formal language theory, a context-free grammar (CFG) is a type of formal grammar, which contains a set of production rules that describe all possible sentences in a given formal language. In Chomsky Normal Form, a context-free grammar $G$ is defined by a 4-tuple $G = (V, \Sigma, R, \Gamma)$ where

- $V$ is a finite set of non-terminal symbols that can be replaced by/expanded to a sequence of symbols.
- $\Sigma$ is a finite set of terminal symbols that represent actual words in a language, which cannot be further expanded.
- $R$ is a finite set of production rules describing the replacement of symbols, typically of the form $A \rightarrow BC$ or $A \rightarrow \alpha$ for $A, B, C \in \Sigma$ and $\alpha \in \Sigma$. A production rule replaces the left-hand side non-terminal symbol by the right-hand side expression. For example, $A \rightarrow BC|\alpha$ means that $A$ can be replaced by either $BC$ or $\alpha$.
- $\Gamma \in V$ is the start symbol (root of the grammar).

Probabilistic Context-Free Grammars (PCFGs) augments CFGs by associating each production rule with a probability. Formally, it is defined by a 5-tuple $G = (V, \Sigma, R, \Gamma, P)$, where $P$ is the set of probabilities on production rules. Figure 2 shows an example probabilistic temporal grammar of the activity “making cereal”.

Given a formal grammar, parsing is the process of analyzing a string of symbols, conforming to the production rules and generating a parse tree. A parse tree represents the syntactic structure of a string according to some context-free grammar. The root node of the tree is the grammar root. Other non-leaf nodes correspond to non-terminals in the grammar, expanded according to grammar production rules (could be expanding a combination or choosing alternatives). The leaf nodes are terminal symbols. All the leaf nodes together form a sentence in the language space described by the grammar.
4 EARLEY PARSER

In this section, we briefly review the original Earley parser\cite{earley}, a classic grammar parsing algorithm with useful concepts that will be extended in the generalized Earley parser. An illustrative example is shown in Fig. 4 to run through the algorithm. We then discuss how the original Earley parser can be applied to event parsing and its drawbacks.

Earley parser is an algorithm for parsing sentences of a given context-free language. In the following descriptions, \(\alpha, \beta,\) and \(\gamma\) represent any string of terminals/nonterminals (including the empty string \(\epsilon\)), \(A\) and \(B\) represent single nonterminals, and \(a\) represents a terminal symbol. We adopt Earley’s dot notation: for production rule of form \(A \rightarrow \alpha\beta\), the notation \(A \rightarrow \alpha \cdot \beta\) means \(\alpha\) has been parsed and \(\beta\) is expected.

Input position \(n\) is defined as the position after accepting the \(n\)th token, and input position \(0\) is the position prior to input. At each input position \(m\), the parser generates a state set \(S(m)\). Each state is a tuple \((A \rightarrow \alpha \cdot \beta, i)\), consisting of:

- The production currently being matched \((A \rightarrow \alpha\beta)\).
- The dot: the current position in that production.
- The position \(i\) in the input at which the matching of this production began: the position of origin.

Seeded with \(S(0)\) containing only the top-level rule, the parser then repeatedly executes three operations: prediction, scanning and completion:

- **Prediction:** for every state in \(S(m)\) of the form \((A \rightarrow \alpha \cdot \beta, i)\), where \(i\) is the origin position as above, add \((B \rightarrow \gamma, m)\) to \(S(m)\) for every production in the grammar with \(B\) on the left-hand side (i.e., \(B \rightarrow \gamma\)).
- **Scanning:** if \(a\) is the next symbol in the input stream, for every state in \(S(m)\) of the form \((A \rightarrow \alpha \cdot A\beta, i)\), add \((A \rightarrow a\beta, i)\) to \(S(m + 1)\).
- **Completion:** for every state in \(S(m)\) of the form \((A \rightarrow \gamma \cdot j)\), find states in \(S(j)\) of the form \((B \rightarrow \alpha \cdot A\beta, i)\) and add \((B \rightarrow \alpha\beta, i)\) to \(S(m)\).

In this process, duplicate states are not added to the state set. These three operations are repeated until no new states can be added to the set. The Earley parser executes in \(O(n^2)\) for unambiguous grammars regarding the string length \(n\), and \(O(n)\) for almost all \(LR(k)\) grammars.

<table>
<thead>
<tr>
<th>state</th>
<th>rule</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S(0))</td>
<td>(\Gamma \rightarrow R) (1.0)</td>
<td>(\Gamma \rightarrow R) (1.0)</td>
</tr>
<tr>
<td>(S(1))</td>
<td>(R \rightarrow N) (0.4)</td>
<td>(N \rightarrow &quot;0&quot;) (0.3)</td>
</tr>
<tr>
<td>(S(2))</td>
<td>(R \rightarrow N + N) (0.6)</td>
<td>(R \rightarrow N + N) (0.6)</td>
</tr>
<tr>
<td>(S(3))</td>
<td>(N \rightarrow 1) (0.7)</td>
<td>(N \rightarrow 1) (0.7)</td>
</tr>
</tbody>
</table>

(a) The input grammar. It contains a root symbol \(\Gamma\), two non-terminal symbols \(R\) and \(N\), three terminal symbols 0, 1 and \(+\). The number to the right of each production rule is the corresponding probability.

Fig. 4: An illustrative example of the original Earley parser.

The original Earley parser inspires a way to do event parsing and prediction from videos\cite{earley}. The video can be first processed by a classifier to be segmented and labeled by actions, thus generating a label sentence. We can apply the Earley parser to parse the sentence to get a partial parse tree. The tree can be partial, since the sentence representing the activity might not be complete. Then action prediction can naturally be accomplished by looking that the open Earley states generated by the “prediction” operation. An example is shown in Fig. 3.
However, this process can be problematic. Since the Earley parser takes symbols as input, it has little guidance to help the segmentation process that happens in the frame level. A more severe problem is that the segmentation and labeling process often generates sentences that are grammatically incorrect, i.e., not in the language space described by the grammar. Thus the sentence cannot be parsed by the parser. In such cases, extra efforts are needed to modify the label sentence. One way to address that is sampling sentences from the language and find the closest alternatives [5]. There also exist work in computational linguistics [50, 51, 52] that address the problem of parsing grammatically incorrect sentences. However, these methods still operates in the symbolic space and does not provide much guidance for frame-level inference. To solve these problems, we propose the generalized Earley parser (detailed in Section 5) that directly takes sequence data as input and generates symbolic parse trees and predictions.

5 Generalized Earley Parser

In this section, we introduce the proposed generalized Earley parser. Instead of taking symbolic sentences as input, we aim to design an algorithm that can parse raw sequence data \( x \) of length \( T \) (e.g., videos or audios) into a sentence \( l \) of labels (e.g., actions or words) of length \( |l| \leq T \), where each label \( k \in \{0, 1, \cdots, K\} \) corresponds to a segment of a sequence.

To achieve that, a classifier (e.g., a neural network) is first applied to each sequence \( x \) to get a \( T \times K \) probability matrix \( y \) (e.g., softmax activations of the neural network), with \( y_k \) representing the probability of frame \( t \) being labeled as \( k \). The proposed generalized Earley parser takes \( y \) as input and outputs the sentence \( l \) that best explains the data according to a grammar \( G \) of Chomsky normal form.

Now we discuss how we generalize the Earley parser to run on the output of a classifier, i.e., the probability matrix. The core idea is to use the original Earley parser to help construct a prefix tree according to the grammar. The best solution is found by performing a heuristic search mass in this tree, where the heuristic is computed based on the probability matrix given by the classifier.

Algorithm 1: The best label sentence \( l \) is returned when the probability of termination is larger than any other prefix probabilities. As long as the parsing and prefix probabilities are computed correctly, it is guaranteed to return the best solution.

The original Earley parser is a special case of the generalized Earley parser. Intuitively, for any input sentence to Earley parser, we can always convert it to one-hot vectors and apply the proposed algorithm. On the other hand, the original Earley parser cannot be applied to segmented words, it measures the probability that \( \exists t \in [0, T] \), the current path \( l \) is the label for frame \( x_{0:t} \). For leaf nodes \( \epsilon \), the heuristic \( p(l|x_{0:T}) \) is a parsing probability; the probability that the current path \( l \) is the label sentence for \( x_{0:T} \). The computation for \( p(l|x_{0:T}) \) and \( p(l_0|0:T) \) are based on the input probability matrix \( y \). The formulation is derived in details in Section 5.2.

This heuristic search generalizes the Earley parser to parse the probability matrix. Specifically, the scan operation in the Earley parser essentially expands a new node in the grammar prefix tree. We organize the states into state sets by the partial sentence (prefix) each state represents. Instead of matching the sentence to the symbolic input, we now process state sets according to their prefix probabilities.

5.1 Parsing Operations

We now describe the details of the parsing operations. Each scan operation will create a new state set \( S(m, n) \in S(m) \), where \( m \) is the length of the scanned string, \( n \) is the total number of the terminals that have been scanned at position \( m \). This can be thought of as creating a new node in the prefix tree, and \( S(m) \) is the set of all created nodes at level \( m \). A priority queue \( q \) is kept for state sets for prefix search. Scan operations will push the newly created set into the queue with priority \( p(l_0) \), where \( l_0 \) is the parsed string of the state being scanned. For brevity, we use \( p(l_0) \) as a shorthand for \( p(l_0|x_{0:T}) \) when describing the algorithm.

Each state is a tuple \( (A \rightarrow \alpha \cdot \beta, i, j, l, p(l_0)) \) augmented from the original Earley parser by adding \( i, j \) to \( l_0 \). Here \( l \) is the parsed string of the state, and \( i, j \) are the indices of the set that this rule originated. The parser then repeatedly executes three operations: prediction, scanning, and completion modified from Earley parser:

- **Prediction:** For every state in \( S(m, n) \) of the form \( (A \rightarrow \alpha \cdot \beta, i, j, l, p(l_0)) \), add \( (B \rightarrow \epsilon, i, j, l, p(l_0)) \) to \( S(m, n) \) for every production in the grammar with \( B \) on the left-hand side.

- **Scanning:** For every state in \( S(m, n) \) of the form \( (A \rightarrow \alpha \cdot \beta, i, j, l, p(l_0)) \), append the new terminal \( a \) to \( l \) and compute the probability \( p(l + a) \). Create a new set \( S(m + 1, n') \) where \( n' \) is the current size of \( S(m + 1) \). Add \( (A \rightarrow a \cdot \beta, i, j, l + a, p(l + a)) \) to \( S(m + 1, n') \). Push \( S(m + 1, n') \) into \( q \) with priority \( p(l + a) \).

- **Completion:** For every state in \( S(m, n) \) of the form \( (A \rightarrow \Gamma, i, j, l, p(l_0)) \), find states in \( S(i, j) \) of the form \( (B \rightarrow \alpha \cdot A \cdot \beta, i', j', l', p(l')) \) and add \( (B \rightarrow \alpha A \cdot \beta, i', j', l, p(l')) \) to \( S(m, n) \).

This parsing process is efficient since we do not need to search through the entire tree. As shown in Figure 6 and Algorithm 1, the best label sentence \( l \) is returned when the probability of termination is larger than any other prefix probabilities. As long as the parsing and prefix probabilities are computed correctly, it is guaranteed to return the best solution.

The original Earley parser is a special case of the generalized Earley parser. Intuitively, for any input sentence to Earley parser, we can always convert it to one-hot vectors and apply the proposed algorithm. On the other hand, the original Earley parser cannot be applied to segmented
TABLE 1: Summary of notations used for parsing & prefix probability formulation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{0:t}$</td>
<td>input frames from time 0 to t</td>
</tr>
<tr>
<td>l</td>
<td>a label sentence</td>
</tr>
<tr>
<td>k</td>
<td>the last label in l</td>
</tr>
<tr>
<td>$l^-$</td>
<td>the label sentence obtained by removing the last label k from the label sentence l</td>
</tr>
<tr>
<td>$y^k_t$</td>
<td>the probability for frame t to be labelled as k</td>
</tr>
<tr>
<td>$p(l</td>
<td>x_{0:t})$</td>
</tr>
<tr>
<td>$p(l^-</td>
<td>x_{0:t})$</td>
</tr>
</tbody>
</table>

one-hot vectors since the labels are often grammatically incorrect. Hence we have the following proposition.

**Proposition 1.** Earley parser is a special case of the generalized Earley parser.

**Proof.** Let $L(G)$ denote the language of grammar $G$, $h(\cdot)$ denote a one-to-one mapping from a label to a one-hot vector. $L(G)$ is the input space for Earley parser. $\forall l \in L(G)$, the generalized Earley parser accepts $h(l)$ as input. Therefore the proposition follows.

Here we emphasize two important distinctions of our method to traditional probabilistic parsers with prefix probabilities. i) In traditional parsers, the prefix probability is the probability of a string being a prefix of some strings generated by a grammar (top-down grammar prior). In our case, the parser computes the bottom-up data likelihood. We further extend this to a posterior that integrates these two abilities. ii) Traditional parsers only maintain a parse tree, while our algorithm maintains both a parse tree and a prefix tree. The introduction of the prefix tree into the parser enables us to efficiently search in the grammar according to a desired heuristic.

### 5.2 Parsing & Prefix Probability Formulation

Table 1 summarizes the notations we use in this section. The parsing probability $p(l|x_{0:T})$ is computed in a dynamic programming fashion. Let $k$ be the last label in $l$. For $t = 0$, the probability is initialized by:

$$p(l|x_0) = \begin{cases} y^k_0 & l \text{ contains only } k, \\ 0 & \text{otherwise}. \end{cases}$$

Let $l^-$ be the label sentence obtained by removing the last label $k$ from the label sentence $l$. For $t > 0$, the last frame $t$ must be classified as $k$. The previous frames can be labeled as either $l$ or $l^-$. Then we have:

$$p(l|x_{0:t}) = y^k_t (p(l|x_{0:t-1}) + p(l^-|x_{0:t-1})), \quad t > 0$$

where $p(l|x_{0:t-1})$ corresponds to the possibility that frame $t-1$ is also labelled as $k$, and $p(l^-|x_{0:t-1})$ accounts for the possibility that label $k$ starts from frame $t$. It is worth mentioning that when $y^k_t$ is wrongly given as 0, the dynamic programming process will have trouble correcting the mistake. Even if $p(l^-|x_{0:t-1})$ is high, the probability $p(l|x_{0:t})$ will be 0. Fortunately, since the softmax function is usually adopted to compute $y$, $y^k_t$ will not be 0 and the solution will be kept for further consideration.

Then we compute the prefix probability $p(l^-|x_{0:T})$ based on $p(l^-|x_{0:t})$. For $l$ to be the prefix, the transition from $l^-$ to $l$ can happen at any frame $t \in \{0, \ldots, T\}$. Once the label $k$ is observed (the transition happens), $l$ becomes the prefix and the rest frames can be labeled arbitrarily. Hence the probability of $l$ being the prefix is:

$$p(l|\ldots|x_{0:T}) = p(l|x_0) + \sum_{t=1}^{T} y^k_t p(l^-|x_{0:t-1}). \quad (3)$$

In practice, the probability $p(l|x_{0:T})$ decreases exponentially as $t$ increases and will soon lead to numeric underflow. To avoid this, the probabilities need to be computed in log space:

$$\log(p(l|x_{0:T})) = \log(y^k_0) + d + \log(\exp(\log(p(l|x_{0:t-1}) - d)) + \exp(\log(p(l^-|x_{0:t-1}) - d), \quad (4)$$

where $d$ is a constant number and is usually set to be max($\log(y^k_t)$, $\log(p(l|x_{0:t-1})$, $\log(p(l^-|x_{0:t-1})$). The time complexity of computing the probabilities is $O(T)$ for each sentence $l$ because $p(l^-|x_{0:t})$ are cached. The worst case complexity of the entire parsing is $O(T|G|)$.

### 5.3 Incorporating Grammar Prior

For PCFGs, we can integrate the grammar prior of the sentence $l$ into the above formulation to obtain a posterior parsing probability. The basic idea is that we can compute a “transition probability” of appending a new symbol to the current sentence. This probability will be multiplied to the parsing probability when we append a new symbol.

To compute a transition probability $p(k|l^-)$, we can first compute the prefix probabilities $p(l^-|G)$ and $p(l^-|G)$ according to the grammar. Then the transition probability is given by:

$$p(k|l^-, G) = \frac{p(l^-|G)}{p(l^-|G)}.$$ \quad (5)

An example is shown in Figure 5 for a better intuition. The computation of this grammar prefix probability will be detailed in Section 5.5.1. There are two important remarks to make here. i) This prior prefix probability is different from the prefix probability based on the likelihood. The prior is the probability that a string is the prefix of a sentence in the
language defined by the grammar, without seeing any data; the likelihood is the probability that a string is the prefix of a video’s label. 2) This grammar-based transition probability is non-Markovian, since the new symbol is conditioned on the entire history string that has a variable length.

Now, incorporating the grammar transition probability, for \( t = 0 \), the probability is initialized by:

\[
p((|x_0, G) \propto \begin{cases} p(k|\epsilon, G) y_k^k & \text{if } l \text{ contains only k,} \\ 0 & \text{otherwise,} \end{cases}
\]

(6)

where \( p(k|\epsilon, G) \) is the probability of appending \( k \) to the empty string \( \epsilon \), which is equivalent to \( p(k|..|G) \) or \( p(l|..|G) \). Notice that the equal sign is replaced by \( \propto \) since the right hand side should be normalized by the prior \( p(x_0) \) to get the correct posterior.

Whenever we append a new symbol to our sentence, we multiply the probability by the transition probability. Hence for \( t > 0 \) we have:

\[
p(l|x_{0:t}, G) \propto y_k^k (p(l|x_{0:t-1}, G) + p(k|l^{-1}, G)p(l^{-1}|x_{0:t-1}, G)).
\]

(7)

Comparing to Eq. 2, we multiply the second term by \( p(k|l^{-1}, G) \) to account for the transition to symbol \( k \).

Finally the posterior probability of \( l \) being the prefix of the label sentence for data \( x \) is:

\[
p(l...|x_{0:T}, G) = p(l|x_0, G) + \sum_{t=1}^{T} p(k|l^{-1}, G)y_k^k p(l^{-1}|x_{0:t-1}, G).
\]

(8)

5.3.1 Grammar Prefix Probability

The derivation of the grammar prefix probability with Earley parser [53] can be achieved by augmenting the Earley states with two additional variables: forward probability \( \mu \) and inner probability \( \nu \). For a state \( S \), the forward probability \( \mu \) is the probability of all parses that lead to \( S \), the inner probability \( \nu \) is the probability of all parses expanded from \( S \). In other words, \( \mu \) is the probability of the prefix before \( S \), and \( \nu \) is the probability of the partial string parsed by \( S \). Assuming that the grammar is not left-recursive, these two terms can be computed efficiently during the Earley parsing process:

- **Prediction.** For \( (A \rightarrow \alpha \cdot B \beta, i, [\mu, \nu]) \Rightarrow (B \rightarrow \gamma, m, [\mu', \nu']) \), the new probabilities are given by

\[
\mu' = \alpha \cdot P(B \rightarrow \gamma), \nu' = P(B \rightarrow \gamma).
\]

- **Scanning.** For \( (A \rightarrow \alpha \cdot a \beta, i, [\mu, \nu]) \Rightarrow (A \rightarrow a a \cdot \beta, i, [\mu', \nu']) \), we have

\[
\mu' = \mu, \nu' = \nu.
\]

- **Completion.** For \( (A \rightarrow \gamma, j, [\mu'', \nu'']) \) and \( (B \rightarrow \alpha \cdot A \beta, i, [\mu', \nu']) \Rightarrow (B \rightarrow a A \beta, i, [\mu', \nu']) \), we have

\[
\mu' = \mu \cdot \nu'', \nu' = \nu \cdot \nu'.
\]

Finally, the prefix probability of a string is given by the sum of forward probabilities over all scanned states. A run-through example of the generalized Earley parser with grammar prior is shown in Figure 6 and in Figure 7.

### Algorithm 1: Generalized Earley Parser

**Input:** Grammar \( G \), probability matrix \( y \)

**Output:** Best label string \( l^* \)

/* For brevity, we denote \( p(; x_{0:i}) \) as \( p(\cdot) \)

/* Initialization */

1. \( S(0, 0) = \{(\Gamma \rightarrow \cdot), 0, 0, e, 1.0)\}\)
2. \( q = priorityQueue()\)
3. \( q.push((1.0, (0, 0, e, S(0, 0)))\)

while \((m, n, t, currentSet) = q.pop()\) do

for \((s = (r, i, j, l, p(l)) \in currentSet)\) do

if \( p(l) > p(l^*); l^* = l \) then \( l^* = l \)

if \( r \) is \((A \rightarrow a \cdot B \beta)\) then // predict

for each \((B \rightarrow \gamma) \in G\) do

\( \gamma' = (B \rightarrow \gamma, i')\)

\( s' = (r', s, l, p(l'))\)

\( S(m, n).add(s')\)

end

end

else if \( r \) is \((A \rightarrow a \cdot a \beta)\) then // scan

\( m' = m + 1, n' = S(m + 1)\)

\( s' = (r', i, j, 1 + a, p(l + a))\)

\( S(m', n').add(s')\)

\( q.push(p(l + a), (m', n', S(m', n')))\)

end

else if \( r \) is \((B \rightarrow \Gamma)\) then // complete

for each \((A \rightarrow a \cdot a B \beta, i', j') \in S(i, j)\) do

\( \gamma' = (A \rightarrow a B \cdot \beta)\)

\( s' = (r', i', j', l, p(l'))\)

\( S(m, n).add(s')\)

end

end

end

if \( p(l^*) > p(l), \forall \text{ un-expanded } l \) then return \( l^* \)

return \( l^* \)

5.4 Segmentation and Labeling

The generalized Earley parser gives us the best grammatically correct label sentence \( l \) to explain the sequence data, which takes all possible segmentations into consideration. Therefore the probability \( p(l|x_{0:T}) \) is the summation of probabilities of all possible segmentations. Let \( p(l|y_{0:e}) \) be the probability of the best segmentation based on the classifier output \( y \) for sentence \( l \). We perform a maximization over different segmentations by dynamic programming to find the best segmentation:

\[
p(l|y_{0:e}) = \max_{b < c} p(l^-|y_{0:b}) \prod_{i=b}^{e} y_i^k
\]

(9)

where \( e \) is the time frame that \( l \) ends and \( b \) is the time frame that \( l^- \) ends. The best segmentation can be obtained by backtracing the above probability. Similar to the previous probabilities, this probability needs to be computed in log space as well. The time complexity of the segmentation and labeling is \( O(T^2) \).
### 5.5 Future Label Prediction

We consider two types of future label predictions: 1) segment-wise prediction that predicts the next segment label at each time $t$, and 2) frame-wise prediction that predicts the labels for the future $\delta t$ frames.

#### 5.5.1 Segment-wise Prediction

Given the parsing result $l$, we can make grammar-based top-down predictions for the next label $z$ to be observed. The predictions are naturally obtained by the predict operation in the generalized Earley parser, and it is inherently an online prediction algorithm.

To predict the next possible symbols at current position $(m, n)$, we search through the states $S(m, n)$ of the form $(X \rightarrow \alpha \cdot z \beta, i, j, l, p(l))$, where the first symbol $z$ after the current position is a terminal node. The predictions $\Sigma$ are then given by the set of all possible $z$:

$$
\Sigma = \{ z : \exists s \in S(m, n), s = (X \rightarrow \alpha \cdot z \beta, i, j, l, p(l)) \}.
$$

The probability of each prediction is then given by the parsing likelihood of the sentence constructed by appending the predicted label $z$ to the current sentence $l$. Assuming that the best prediction corresponds to the best parsing result, the goal is to find the best prediction $z^*$ that maximizes the following conditional probability as parsing likelihood:

$$
z^* = \arg \max_{z \in \Sigma} p(z, l|G).
$$

For a grammatically complete sentence $u$, the parsing likelihood is simply the Viterbi likelihood [54] given by the probabilistic context-free grammar. For an incomplete sentence $l$ of length $|l|$, the parsing likelihood is given by the grammar prefix probability. Hence they are both the forward probability computed in Section 5.3.1. We can also integrate top-down and bottom-up inference for segment-wise prediction. A classifier can be trained to predict the next segment label, and it can be combined with the grammar prior probability for better predictions.

#### 5.5.2 Frame-wise Prediction

Frame-wise future label prediction is rather straightforward using the generalized Earley parser. We first run activity detection on the input videos, and we sample the duration of the current action. Based on the segment-wise prediction, we can further sample the duration for future segments, thus obtaining frame-wise future predictions according to the prediction range.

#### 5.5.3 Maximum Likelihood Estimation for Prediction

We are interested in finding the best grammar and classifier that give us the most accurate segment-wise predictions.
Fig. 6: It performs a heuristic search in the prefix tree according to the prefix/parsing probability. It iteratively expands the tree and computes the probabilities as it expands the tree. The search ends when it hits a parsing terminal $e$. The paths in bold indicate the best candidates at each search step.

The best classifier minimizes the second term of Eq. 13

$$f^* = \arg\min_f \sum_{(x,l,z) \in D} \log(p(l|x))$$

$$\approx \arg\min_f \sum_{(x,y) \in D} \sum_k y_k \log(\tilde{y}_k),$$

where $p(l|x)$ can be maximized by the CTC loss [57]. In practice, it can be substituted by the commonly adopted cross entropy loss for efficiency. Therefore we can directly apply generalized Earley parser to outputs of general detectors/classifiers for parsing and prediction.

6 Human Activity Parsing and Prediction

We evaluate our method on the task of human activity detection and prediction. We present and discuss our experiment results on three datasets, CAD-120 [2], Watch-n-Patch [3], and Breakfast [4], for comparisons with state-of-the-art methods and evaluation of the robustness of our approach. CAD-120 is the dataset that most existing prediction algorithms are evaluated on. It contains videos of daily activities that are long sequences of sub-activities. Watch-n-Patch is a daily activity dataset that features forgotten actions. Breakfast is a dataset that contains long videos of daily cooking activities. Results show that our method performs well on both activity detection and activity prediction.

6.1 Grammar Induction

In both experiments, we used a modified version of the ADIOS (automatic distillation of structure) [55] grammar induction algorithm to learn the event grammar. The algorithm learns the production rules by generating significant patterns and equivalent classes. The significant patterns are selected according to a context-sensitive criterion defined regarding local flow quantities in the graph: two probabilities are defined over a search path. One is the right-moving ratio of fan-through (through-going flux of path) to fan-in (incoming flux of paths). The other one, similarly, is the left-going ratio of fan-through to fan-in. The criterion is described in detail in [55].
6.2 Experiment on CAD-120 Dataset

**Dataset.** The CAD-120 dataset [2] is a standard dataset for human activity prediction. It contains 120 RGB-D videos of four different subjects performing 10 high-level activities, where each high-level activity was performed three times with different objects. It includes a total of 61,585 total video frames. Each video is a sequence of sub-activities involving 10 different sub-activity labels. The videos vary from subject to subject regarding the lengths and orders of the sub-activities as well as the way they executed the task.

**Evaluation metrics.** We use the following metrics to evaluate and compare the algorithms. 1) Frame-wise detection accuracy of sub-activity labels for all frames. 2) Frame-wise (future 3s) online prediction accuracy. We compute the frame-wise accuracy of prediction of the sub-activity labels of the future 3s (using the frame rate of 14Hz as reported in [2]). The predictions are made online at each frame $t$, i.e., the algorithms only sees frame 0 to $t$ and predicts the labels of frame $t$ to $t + 6t$. 3) Segment-wise online prediction accuracy. At each frame $t$, the algorithm predicts the sub-activity label of the next video segment.

We consider the overall micro accuracy (P/R), macro precision, macro recall and macro F1 score for all evaluation metrics. Micro accuracy is the percentage of correctly classified labels. Macro precision and recall are the average of precision and recall respectively for all classes.

**Comparative methods.** We compare the results for the following methods:

1) **KGS [2]**: a Markov random field model where the nodes represent objects and sub-activities, and the edges represent the spatial-temporal relationships. Future frames are predicted based on the transition probabilities given the inferred label of the last frame.

2) **Anticipatory temporal CRF (ATCRF) [48]**: one of the state-of-the-art prediction methods. It is an anticipatory conditional random field that models the spatial-temporal relations through object affordances. Future frames are predicting by sampling a spatial-temporal graph.

3) **ST-AOG [5]**: a spatial-temporal And-Or graph (ST-AOG) that uses a symbolic context-free grammar to model activity sequences. This sets up a comparison between our proposed method and methods that use traditional probabilistic parsers. Since traditional parsers operate on symbolic data, extra efforts need to be done first to extract symbols from sequence data. In this comparative method, the videos are first segmented and labeled by classifiers; the predictions are then made by the original Earley parser.

4) **Bidirectional LSTM (Bi-LSTM):** a simple frame-wise detection classifier. It outputs a sub-activity label for every input frame feature.

5) **LSTM:** a simple prediction classifier. We do not use the Bi-LSTM for prediction since the model should not see the future frames. For the future 3s (frame-wise) prediction, the LSTM is trained to output the label for frame $t + 3$ for an input frame at time $t$. For future segment prediction, it outputs the label of the next segment for an input frame.

6) **Bi-LSTM/LSTM + generalized Earley parser:** the proposed generalized Earley parser applied to the classifier output for detection and prediction. The predictions for the next segments are made according to Section 5.5. The lengths of unobserved segments are sampled from a log-normal distribution for the future 3s prediction.

**Implementation details.** We use the same Bi-LSTM as the base classifier for detection task and LSTM as the base classifier for prediction tasks on all three datasets. For both

---

**TABLE 2: Detection results on CAD-120.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Micro P/R</th>
<th>Macro P/R</th>
<th>Recall</th>
<th>F1-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>KGS [2]</td>
<td>68.2</td>
<td>71.1</td>
<td>62.2</td>
<td>66.4</td>
</tr>
<tr>
<td>ATCRF [48]</td>
<td>70.3</td>
<td>74.8</td>
<td>66.2</td>
<td>70.2</td>
</tr>
<tr>
<td>ST-AOG + Earley [5]</td>
<td>76.5</td>
<td>77.0</td>
<td>75.2</td>
<td>76.1</td>
</tr>
<tr>
<td>Bi-LSTM</td>
<td>76.2</td>
<td>78.5</td>
<td>74.5</td>
<td>74.9</td>
</tr>
<tr>
<td>Bi-LSTM + Generalized Earley</td>
<td>79.4</td>
<td>87.4</td>
<td>77.0</td>
<td>79.7</td>
</tr>
</tbody>
</table>

**TABLE 3: Future 3s prediction results on CAD-120.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Micro P/R</th>
<th>Macro P/R</th>
<th>Recall</th>
<th>F1-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>KGS [2]</td>
<td>28.6</td>
<td>–</td>
<td>–</td>
<td>11.1</td>
</tr>
<tr>
<td>ATCRF [48]</td>
<td>49.6</td>
<td>–</td>
<td>–</td>
<td>40.6</td>
</tr>
<tr>
<td>ST-AOG + Earley [5]</td>
<td>55.2</td>
<td>56.5</td>
<td>56.6</td>
<td>56.6</td>
</tr>
<tr>
<td>LSTM</td>
<td>49.4</td>
<td>40.9</td>
<td>37.3</td>
<td>37.8</td>
</tr>
<tr>
<td>LSTM + Generalized Earley</td>
<td>57.1</td>
<td>52.3</td>
<td>54.1</td>
<td>52.3</td>
</tr>
</tbody>
</table>

**TABLE 4: Segment prediction results on CAD-120.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Micro P/R</th>
<th>Macro P/R</th>
<th>Recall</th>
<th>F1-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST-AOG + Earley [5]</td>
<td>54.3</td>
<td>61.4</td>
<td>39.2</td>
<td>45.4</td>
</tr>
<tr>
<td>LSTM</td>
<td>52.8</td>
<td>52.5</td>
<td>52.8</td>
<td>47.6</td>
</tr>
<tr>
<td>LSTM + Generalized Earley</td>
<td>70.6</td>
<td>72.1</td>
<td>70.6</td>
<td>70.1</td>
</tr>
</tbody>
</table>
models, we used a 2 layer LSTM backbone with hidden size 256 and added bidirectional propagation for the Bi-LSTM model. For training, we use a Adam optimizer with learning rate $1 \times 10^{-3}$ and set weight decay as 0.8 for every 20 epochs.

**Feature extraction.** All methods in the experiment use the same publicly available features from KGS [2]. These features include the human skeleton features and human-object interaction features for each frame. The human skeleton features are location and distance features (relative to the subjects head location) for all upper-skeleton joints of a subject. The human-object features are spatial-temporal, containing the distances between object centroids and skeleton joint locations as well as the temporal changes.

**Experiment results.** We follow the convention in KGS [2] to train on three subjects and test on a new subject with a 4-fold validation. The results for the three evaluation metrics are summarized in Table 2, Table 3, and Table 4 respectively. Figure 8 shows the confusion matrices for the two prediction tasks.

Our method outperforms the comparative methods on all three tasks. Specifically, the generalized Earley parser on top of a Bi-LSTM performs better than ST-AOG, while ST-AOG outperforms the Bi-LSTM. More discussions are highlighted in Section 6.5.

### 6.3 Experiment on Watch-n-Patch Dataset

**Dataset.** Watch-n-Patch [3] is an RGB-D dataset that features forgotten actions. For example, a subject might fetch milk from a fridge, pour milk, and leave. The typical action “putting the milk back into the fridge” is forgotten. The dataset contains 458 videos with a total length of about 230 minutes, in which people forgot actions in 222 videos. Each video in the dataset contains 2-7 actions interacted with different objects. 7 subjects are asked to perform daily activities in 8 offices and 5 kitchens with complex backgrounds. It consists of 21 types of fully annotated actions (10 in the office, 11 in the kitchen) interacted with 23 types of objects.

**Feature extraction.** We extract the same features as described in [3] for all methods. Similar to the previous experiment, the features are composed of skeleton features

<table>
<thead>
<tr>
<th>Method</th>
<th>Micro P/R</th>
<th>Macro P/R</th>
<th>Macro Recall</th>
<th>Macro F1-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST-AOG + Earley [5]</td>
<td>79.3</td>
<td>71.5</td>
<td>73.5</td>
<td>71.9</td>
</tr>
<tr>
<td>Bi-LSTM</td>
<td>84.0</td>
<td>79.7</td>
<td>82.2</td>
<td>80.3</td>
</tr>
<tr>
<td>Bi-LSTM + Generalized Earley</td>
<td>84.8</td>
<td>80.7</td>
<td>83.4</td>
<td>81.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>Micro P/R</th>
<th>Macro P/R</th>
<th>Macro Recall</th>
<th>Macro F1-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST-AOG + Earley [5]</td>
<td>48.9</td>
<td>43.1</td>
<td>39.3</td>
<td>39.3</td>
</tr>
<tr>
<td>LSTM</td>
<td>43.9</td>
<td>28.3</td>
<td>26.6</td>
<td>24.9</td>
</tr>
<tr>
<td>LSTM + Generalized Earley</td>
<td>58.7</td>
<td>50.5</td>
<td>49.9</td>
<td>49.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>Micro P/R</th>
<th>Macro P/R</th>
<th>Macro Recall</th>
<th>Macro F1-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST-AOG + Earley [5]</td>
<td>29.4</td>
<td>28.5</td>
<td>18.9</td>
<td>19.9</td>
</tr>
<tr>
<td>LSTM</td>
<td>44.6</td>
<td>43.6</td>
<td>44.6</td>
<td>40.4</td>
</tr>
<tr>
<td>LSTM + Generalized Earley</td>
<td>49.5</td>
<td>50.1</td>
<td>49.4</td>
<td>45.5</td>
</tr>
</tbody>
</table>
TABLE 8: Detection results on Breakfast.

<table>
<thead>
<tr>
<th>Method</th>
<th>Micro P/R</th>
<th>Macro Prec.</th>
<th>Recall F1-score</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOG+HTK [4]</td>
<td>28.8</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>ED-TCN [59]</td>
<td>43.3</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>TCFPN [60]</td>
<td>52.0</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Fisher+HTK [61]</td>
<td>56.3</td>
<td>38.1</td>
<td>–</td>
</tr>
<tr>
<td>Bi-LSTM</td>
<td>45.6</td>
<td>29.2</td>
<td>25.4</td>
</tr>
<tr>
<td>Bi-LSTM + Generalized Earley</td>
<td>59.7</td>
<td>45.8</td>
<td>36.3</td>
</tr>
</tbody>
</table>

*The results for [59] is obtained from [60].

and human-object interaction features extracted from RGB-D images. The skeleton features include angles between connected parts, the change of joint positions and angles from previous frames. Each frame is segmented into super-pixels, and foreground masks are detected. We extract features from the image segments with more than 50% in the foreground mask and within a distance to the human hand joints in both 3D points and 2D pixels. Six kernel descriptors [58] are extracted from these image segments: gradient, color, local binary pattern, depth gradient, spin, surface normals, and KPCA/self-similarity.

**Experiment results.** We use the same evaluation metrics as the previous experiment and compare our method to ST-AOG [5] and Bi-LSTM. We use the train/test split in [3]. The results for the three evaluation metrics are summarized in Table 5, Table 6, and Table 7, respectively. Our method slightly improves the detection results over the Bi-LSTM outputs, and outperforms the other methods on both prediction tasks. In general, the algorithms make better predictions on CAD-120, since Watch-n-Patch features forgotten actions and the behaviors are more unpredictable. Figure 11 shows some qualitative results, and more details are discussed in Section 6.5.

6.4 Experiment on Breakfast Dataset

**Dataset.** Breakfast [4] is a dataset of daily cooking activities. The dataset includes 52 unique participants, each conducting 10 distinct cooking activities captured in 18 different kitchens. It has \sim 77 hours of video footage containing different camera views (3 to 5 depending on the location). For data annotations, 48 different coarse action units are identified with 11,267 samples (segments) in total including \sim 3,600 silence samples.

**Comparative methods.** Besides Bi-LSTM, we compare Bi-LSM + Generalized Earley with state-of-art methods for activity detection on the Breakfast dataset:

1) HOG+HTK [4]: a grammar-based method. The authors proposed to use the hidden Markov model (HMM) for modeling individual action units in the sequence recognition problem. These action units then form the building blocks to model complex human activities as sentences using an action grammar. A speech recognition engine (the HTK toolkit [62]) is used for recognition on top of the extracted HOG+HTK features [2].

2) ED-TCN [59]: an end-to-end method. Encoder-Decoder Temporal Convolutional Network is proposed to tackle the action classification problem. Under the model’s setting, predictions at each frame are a function of a fixed-length period of time, which is referred to by the authors as the receptive field.

3) TCFPN [60]: one of the end-to-end state-of-the-art methods. The Temporal Convolutional Feature Pyramid retains the encoder-decoder architecture and adapt the lateral connection mechanism proposed in Feature Pyramid networks to the task of action segmentation.

4) Fisher+HTK [61]: one of the grammar-based state-of-the-art methods. Similar to [4], the action units are modeled by HMM, and high-level activities are modeled by action grammars. The main difference is the feature (Fisher kernels [63]) proposed for action recognition.

**Experiment results.** To eliminate the factors of feature extraction for fair comparison, we use the pre-computed feature provided by [61] to train the underlying Bi-LSTM.
classifier. Figure 9 shows the qualitative results of activity detection on the Breakfast dataset. The quantitative results (Table 8) show that a simple Bi-LSTM is far from state-of-the-art methods (an absolute difference of 10.7%). Our full algorithm Bi-LSTM + Generalized Earley improves the absolute performance by 14.1%, and outperforms the state-of-the-art by 3.6%. This shows that our explicit grammar regularization is effective in correcting the mistakes of the underlying classifier. Although the underlying classifier is simple, it is able to perform well in the activity detection task well.

6.5 Discussion

How does the performance change with respect to the prediction duration? Figure 10 shows how the frame-wise prediction accuracy changes when the prediction duration varies on the CAD-120 dataset. In the CAD-120 dataset, the average duration of a sub-activity is around 3.6 seconds. Hence a prediction duration of 10 seconds is over two to three sub-activities. When the prediction duration increases, the performance of both approaches drop, but we can see that the generalized Earley parser still improves the performance of the underlying classifier.

Can we reduce the running time for the generalized Earley parser? When running the generalized Earley parser for detection, the most time-consuming step is the segmentation by dynamic programming. As discussed in Section 5.4, the complexity for segmentation and labeling is \(O(T^2)\) if we run the complete algorithm. One approximate solution is to run the algorithm on a sub-sampled video. This would reduce the running time quadratically. The potential effect of sub-sampling is two-fold: 1) certain information is dropped and could hurt the performance, whereas 2) given the length of the activities are generally long, sub-sampling can serve as a weak duration prior that might improve performance. We study the effect of sub-sampling the video, and Figure 12 shows the performance under different sample rates for both the Bi-LSTM and Bi-LSTM + generalized Earley parser.

As we can see from Figure 12, our proposed model outperforms Bi-LSTM models under all sample rates. The results show a slight increase for both models when the sample rate increase. We argue that this matches with the second hypothesis where the added sub-sampling procedure can be seen as adding basic duration constraints. We also conjecture that the sub-sampled features become more distinguishable which further contributes to the overall performance improvement.

How does the generalized Earley parser refine the classifier’s output predictions? When the classifier outputs violate the grammar, two types of refinements occur: i) correction and deletion of wrong labels as shown in Figure 11a; ii) insertion of new labels as shown in Figure 11b. The inserted segments are usually very short to accommodate both the grammar and the classifier outputs. Most boundaries of the refined results are well aligned with the classifier outputs.

Why do we use two metrics for future prediction? The future 3s prediction is a standard evaluation criterion set up by KGS and ATCRF. However, this criterion does not tell how well the algorithm predicts the next segment label. i) At any time frame, part of the future 3s involves the current sub-activity for most of the times. ii) If the predicted length of the current sub-activity is inaccurate, the frame-wise inaccuracy drops proportionally, even when the future segment label prediction is accurate. Therefore we also compare against the future segment label prediction because it is invariant to variations in activity lengths.

How well does the generalized Earley parser perform for activity detection and prediction? In some cases it slightly improves over the classifier outputs for detection (Table 5), but still significantly outperforms the classifier for predictions (Table 6 and Table 7). The modifications on classifier outputs (corrections and insertions in Figure 11) are minor but important to make the sentences grammatically correct, thus high-quality predictions can be made.

How useful is the grammar for activity modeling? From Table 3, Table 4, Table 6 and Table 7 we can see that both ST-AOG and generalized Earley parser outperforms Bi-LSTM for prediction. Prediction algorithms need to give different outputs for similar inputs based on the observation history. Hence the non-Markovian property of grammars is useful for activity modeling, especially for future prediction.

How robust is the generalized Earley parser? Comparing Table 4 and Table 7 we can see that there is a performance drop when the action sequences are more unpredictable (in the Watch-n-Patch dataset). But it is capable of
improving over the noisy classifier inputs and significantly outperforms the other methods. It is also robust in the sense that it can always find the best sentence in a given language that best explains the classifier outputs.

7 Conclusion

We proposed a generalized Earley parser for parsing sequence data according to symbolic grammars. Detections and predictions are made by the parser given the probabilistic outputs from any classifier. We are optimistic about and interested in further applications of the generalized Earley parser. In general, we believe this is a step towards the goal of integrating the connectionist and symbolic approaches.

Acknowledgments

The authors would like to thank Professor Ying Nian Wu from UCLA Statistics Department for helpful comments on this work. The work reported herein is supported by DARPA XAI N66001-17-2-4029, ONR MURI N00014-16-1-2007, and N66001-17-2-3602.

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